CSci 435: Formal Languages and Automata

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**Home Assignment 7: 50 points + 30 points (optional)**

Q1.[10] Prove if the following languages are CFL or not.

If L is a CFL, give its CFG. Otherwise, prove it by Pumping Lemma.

If any closure property of CFL is applicable, apply them to simplify it before its proof.

1. [10] L = {*wwRw* | *w* ∈ {*a, b*}\*} is not CFL.

We assume that the language is context free and we assume that there is a push down automata for the language

We take a string in the language w = ambmbmamambm which is a valid string from L

Dividing the string into 5 parts we get u = a v = b, x = ba, y = a, z = b

By pumping lemma any value of i such that uvixyiz belongs to L then L is context free

Assuming i is 2 the string abbbaaab is formed which would not belong in the language proving by contradiction that the language is not a context free language.

1. [10, optional] L = { *an*| *n* is a prime number } is not CFL.

Primes: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29…

We assume that the language is context free and we assume that there is a PDA for the language

We take the string aaaaaaa or a7 and divide the string into 5 parts where u = a v =aa x = a y = aa z = a

By pumping lemma any value of I such that uvixyiz belongs to L then L is context free

Assuming i is 2 the string a aaaa a aaaa a is formed equaling a11 which is part of the language

Assuming i is 3 the string a aaaaaa a aaaaaa a is formed equaling a15 which is not part of the language which proves by contradiction that the language is not a context free language.

Q2. [20] Prove that the following languages are linear or not.

If L is linear, give the linear-CFG for L. Otherwise, prove it by Pumping Lemma for a Linear-CFL.

1. [10] L = {*anbnambm* | *n* ≥ 0, *m* ≥ 0} is CFL, but not linear.

Hint: Give its CFG to show CFL, then prove it’s not linear by pumping lemma.

The language is not a CFG

S → A | aSb | λ

A → aAa | λ

Assuming that L is linear any string in L such that S <= p can be divided into xyz.

String S = abab

By pumping lemma any value of i such that xyiz belongs to L then L is linear

Assuming i is 2 the string produced is ababab which breaks the conditions of L proving by contradiction that L is not linear

[10] L = { *anbmcn* | *n, m* ≥ 0 } ∪ { *anbncm* | *n, m* ≥ 0 } is linear.

The CFG of this language proves that it is linear.

S → AaScBb

A → aA | λ

B → bB | λ

Q3. [10] For the CFG G1 and G2, where

G1: S1 → S1A | *ab*, A → *aa*A | *aa*,

G2: S2 → S2C | *ab*, C → bC | *a*B, B → bbB | bb,

find Context-Free Grammar G’ = (V’, T’, S’, P’) for the following language, L(G1)⋅L(G2).

The union of both grammars creates L(G1\*G2) with productions P

S → S1S2 | λ

S1 → S1A1 | ab

A1 → aaA1 | aa

S2 → S2C2 | ab

C2 → bC2 | aB2

B2 → bbB2 | bb

Q4. [10] Prove the following properties. – See the examples of proof in the slide/textbook.

1. [10] The family of CFLs is closed under reversal.

Hint: For a CFG G=(V, T, S, P) s.t. L(G) = L, construct a CFG G’, s.t. L(G’) = LR.

Then, show that wR ∈ L(G’) iff wR ∈ LR.

To prove closure we would need to make a new grammar G’ such that S’ is created with λ production and all other productions with reversed sides so X → x would become x → X. after all of the productions are created or reversed we prove a string SR is fits L(G’) and S fits L(G) if a string is generated with G using the methods in G’ the strings will be exact reverse copies of strings generated by G indicating that G’ can generate the reverse of S meaning L(G’) = LR this proves that CFL are closed under reversal.

1. [10, optional] The family of CFLs is not closed under complement. Give an example for it.
2. [10, optional] If L1 is linear and L2 is regular, L1⋅L2 is a linear language.